

# Frequency- and transverse wave-vector-dependent spin Hall conductivity in two-dimensional electron gas with disorder

Sudhansu S. Mandal and Ankur Sensharma\*

Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Kolkata 700 032, India  
(Received 24 July 2008; revised manuscript received 30 September 2008; published 12 November 2008)

We determine wave number  $q$  and frequency  $\omega$  dependent spin Hall conductivities  $\sigma_{yx}^s(q, \omega)$  for a disordered two-dimensional electron system with Rashba spin-orbit interaction when  $\mathbf{q}$  is *transverse* to the electric field. Both the conventional definition of spin current and its new definition which takes care of the conservation of spins have been considered. The spin Hall conductivities for both of these definitions are qualitatively similar.  $\sigma_{yx}^s(q, \omega)$  is zero at  $q=0, \omega=0$  and is maximum at  $q=0$  and at small but finite  $\omega$  whose value depends on different parameters of the system. Interestingly for  $\omega \rightarrow 0$ ,  $\sigma_{yx}^s(q)$  resonates when  $\Lambda \approx L_{so}$  which are the wavelength ( $\Lambda = 2\pi/q$ ) of the electric field's spatial variation and the length for one cycle of spin precession, respectively. The sign of the out-of-plane component of the electrons' spin flips when the sign of electric field changes due to its spatial variation along transverse direction. It changes the mode of spin precession from clockwise to counterclockwise or vice versa, and consequently a finite spin Hall current flows in the bulk of the system.

DOI: 10.1103/PhysRevB.78.205313

PACS number(s): 72.25.-b, 71.70.Ej

## I. INTRODUCTION

One of the primary goals today in spin-based electronics<sup>1</sup> is the generation of spin current. Recent realization of spin Hall effect (SHE) (Refs. 2–4) in semiconductor systems is certainly a very significant achievement in this direction. This is a phenomenon for electrical generation of spin: a charge current along its transverse direction induces a spin current whose polarization is perpendicular to the plane formed by these two currents. This phenomenon was predicted<sup>5–7</sup> long ago due to the spin-asymmetric “skew-scattering” mechanism and spin-dependent “side-jump” mechanism which are collectively called *extrinsic* mechanisms because the spin-orbit interaction (SOI) is disordered in this case. This mechanism is also responsible for anomalous Hall effect (AHE) (Refs. 8 and 9) in ferromagnets. However uniform (pure) SOI which is *intrinsic*<sup>10</sup> also causes AHE. Similar intrinsic mechanism due to the SOI in hole-doped semiconductors<sup>11</sup> and two-dimensional electron gas (2DEG) (Ref. 12) in semiconductor heterostructures have been predicted to give rise to nondissipative spin Hall conductivity (SHC).

The spin accumulation observed in  $n$ -doped GaAs (Ref. 2) is believed to be extrinsic in origin because of small spin accumulation and its directional independence on the electric field. On the other hand spin accumulation in two-dimensional hole gas (2DHG) is large<sup>3</sup> and hence is suggested to be intrinsic in origin. These experiments do not measure the spin voltage or the spin current, however the technique developed in observing charge accumulation<sup>13</sup> at the transverse edge due to spin current which is called inverse SHE could be useful in measuring spin Hall current. Nevertheless estimated SHC from the observation of spin accumulation<sup>2</sup> in  $n$ -doped GaAs is in good agreement with the calculated<sup>14,15</sup> SHC in extrinsic mechanisms. An effective two band cubic (in momentum) Rashba model describes the electronic states in 2DHG well. Even in presence of disorder, the SHC in 2DHG has nonzero<sup>16</sup> intrinsic contribution.

The responsible mechanism for SHE in 2DEG which we study here is particularly not clear yet. The electronic states in the 2DEG formed in semiconductor heterostructures can be well described by the Hamiltonian

$$H_0 = \frac{\mathbf{k}^2}{2m} \sigma_0 + \lambda(k_y \sigma_1 - k_x \sigma_2), \quad (1)$$

where  $\mathbf{k}$  and  $m$  are the momentum and mass of the electrons, respectively,  $\lambda$  is the Rashba spin-orbit coupling strength,<sup>17</sup>  $\sigma_0$  is the unit matrix, and  $\sigma_i$  are the Pauli matrices. (We have set the unit  $\hbar = c = 1$ .) Sinova *et al.*<sup>12</sup> predicted universal SHC  $\sigma_{yx}^s = e/8\pi$  for such systems using conventional definition of spin current  $\hat{\mathbf{j}}^\alpha = \frac{1}{2} \{s_\alpha, \hat{\mathbf{v}}_k\}$ , where group velocity  $\hat{\mathbf{v}}_k = \nabla_k H_0$  and  $s_\alpha = \sigma_\alpha / 2$  is the  $\alpha$ th ( $\alpha = 1-3$ ) component of spin. However after a prolonged debate,<sup>18–28</sup> the consensus arising from various methods of calculations is that  $\sigma_{yx}^s = 0$  in presence of disorder no matter what its strength is. These studies include calculation of vertex correction<sup>19</sup> in Born approximation, using Keldysh formalism<sup>20,21,27</sup> for any value of lifetime  $\tau$ , using Kubo formula analytically<sup>22,23,28</sup> and numerically,<sup>25,26</sup> and Boltzmann transport equation approach.<sup>24</sup> The equation of motion for spin projected on the plane is  $\partial_t(\sigma_1, \sigma_2) = -4m\lambda(\hat{j}_x^3, \hat{j}_y^3)$ . A very unique feature<sup>18,22</sup> of the linear Rashba model is that  $\partial_t \sigma_2$  is proportional to  $\hat{j}_y^3$ . It suggests zero-spin Hall current in the steady state.<sup>22</sup> This simple argument describes the vanishing  $\sigma_{yx}^s$  for such systems. A similar argument also describes spin-spin Hall current<sup>28</sup> since  $j_x^1 = -j_y^2$  for steady state derivable from the equation  $\partial_t \sigma_3 = 4m\lambda(\hat{j}_x^1 + \hat{j}_y^2)$ . All these results are obtained from the above conventional definition of  $\hat{\mathbf{j}}^\alpha$  which is not conserved. The new definition of conserved spin current proposed by Shi *et al.*<sup>29</sup> gives rise to vanishing total  $\sigma_{yx}^s$  for short-ranged  $\delta(\mathbf{r})$  impurity potential and for long-ranged potential up to first-order Born approximation.<sup>27</sup>

Is then the intrinsic mechanism really absent for spin Hall effect in 2DEG? In a disordered 2DEG, an in-plane applied

magnetic field may lead to the nonvanishing intrinsic SHC (Ref. 30) due to Zeeman coupling. Further the interplay of Zeeman coupling with different spin-orbit interactions may also lead to finite SHC (Ref. 31) in a pure system. In this paper, we calculate SHC using Kubo formula at finite frequency  $\omega$  and momentum  $\mathbf{q}$  transverse to the applied electric field within the intrinsic mechanism in a disordered 2DEG with no applied magnetic field. We find that even in the static limit, SHC is nonvanishing and hence the presence of “intrinsic” mechanism for spin Hall effect in 2DEG is demonstrated.

The paper is organized as follows. In Sec. II, we calculate frequency and transverse momentum-dependent SHC in a disordered 2DEG with Rashba SOI using Kubo formula with the conventional definition of spin current. The contribution of spin torque to the SHC is also calculated and this contribution, shown in Sec. III, is qualitatively similar. We find that SHC is reasonably high at some range of frequencies and momenta. Particularly interesting case is for static but spatially varying electric field. The SHC resonates when the wavelength of the spatial variation of the electric field matches with the spin precession length. A simple mechanism for this “anomalous” spin Hall current in 2DEG is described in Sec. IV. Section V is devoted for an experimental proposal to test this mechanism, discussion, and summary.

## II. SPIN HALL CONDUCTIVITY

The spin Hall current for an electric field  $\mathbf{E}(\mathbf{q}, \omega)$  at the wave vector  $\mathbf{q}$  transverse to the direction of  $\mathbf{E}$  and at the frequency  $\omega$ ,  $\mathbf{j}_y^s(q, \omega) = \sigma_{yx}^s(q, \omega) E_x(q, \omega)$ . The spin Hall conductivity is nonlocal; spin current at a position  $\mathbf{r}$  depends on the electric field surrounding it  $\mathbf{j}_y^s(\mathbf{r}, \omega) = \int d\mathbf{r}' \sigma_{yx}^s(|\mathbf{r} - \mathbf{r}'|, \omega) E_x(\mathbf{r}', \omega)$ . Using Kubo formula,<sup>32</sup> we find the transverse spin Hall conductivity

$$\sigma_{yx}^s(\mathbf{q}, \omega) = \frac{1}{2\pi} \Re \text{Tr} \left[ \int \frac{d\mathbf{k}}{(2\pi)^2} \hat{j}_y^s \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) \hat{G}_{\mathbf{k}}^A(0) \times \left\{ \hat{j}_x^s \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) + \hat{j}_x^s(\mathbf{q}, \omega) \right\} \hat{G}_{\mathbf{k}+\mathbf{q}}^R(\omega) \right], \quad (2)$$

with

$$\hat{j}_x^s = \frac{1}{m\tau} \int \frac{d\mathbf{k}'}{(2\pi)^2} \hat{G}_{\mathbf{k}'}^A(0) \left\{ \hat{j}_x^s \left( \mathbf{k}' + \frac{\mathbf{q}}{2} \right) + \hat{j}_x^s \right\} \hat{G}_{\mathbf{k}'+\mathbf{q}}^R(\omega). \quad (3)$$

Here retarded (advanced) Green's function for an energy  $\epsilon$  can be written as

$$\hat{G}_{\mathbf{k}}^{R,A}(\epsilon) = \frac{1}{2} \sum_{s=\pm} \frac{\sigma_0 + s(k_y \sigma_1 - k_x \sigma_2)/|\mathbf{k}|}{\epsilon - \xi_{\mathbf{k}}^s \pm \frac{i}{2\tau}}. \quad (4)$$

Equations (2) and (3) together describe sum over infinite series of ladder diagrams. We solve the matrix Eq. (3) numerically and then using Eq. (2) we calculate  $\sigma_{yx}^s(q, \omega)$  when  $\mathbf{E} \parallel \mathbf{e}_x$  and  $\mathbf{q} \parallel \mathbf{e}_y$ , i.e., transverse  $\sigma_{yx}^s(q)$ . For a system with Fermi energy  $\epsilon_F$  and spin-splitting energy  $2\lambda k_F$ , with  $k_F$  being the Fermi momentum, we choose two parameters  $\Delta$

$= \epsilon_F \tau$  and  $\delta = 2\lambda k_F \tau$  comparing to disorder broadening  $1/\tau$ .

We show  $\sigma_{yx}^s(q, \omega)$  for  $\Delta = 10$  and  $20$  and  $\delta = 0.4$  and  $0.8$  in Fig. 1. The standard resonances occur at finite  $\omega$  and at zero or very low value of  $q$ . The maximum value of  $\sigma_{yx}^s(q, \omega)$  is almost proportional to  $\delta^2$  and it decreases with the increase of  $\Delta$ . One common interesting feature for different combination of the parameters to notice is that the value of  $\sigma_{yx}^s(q, \omega)$  is not small for  $\omega \rightarrow 0$  and  $q/(2m\lambda) \approx 1.0$ . Figure 2 shows  $\sigma_{yx}^s(q, 0)$  for  $\Delta = 10$  and  $\delta = 0.1, 0.4, 0.8,$  and  $1.2$ . These choices of  $\delta$  correspond to  $\ell < L_{so}$ , where  $L_{so} = \pi/(m\lambda)$  being the length traversed by an electron while its spin precesses by one cycle. We have checked that  $\sigma_{yx}^s(q, 0)$  is independent of  $\Delta$  for a wide range of  $\Delta > 1$  while  $\delta$  is fixed and is almost proportional to  $\delta^2$ .  $\sigma_{yx}^s$  is zero at  $q=0$  as we know from various calculations<sup>19-28</sup> and then it gradually increases with  $q$  and forms a peak around  $q \approx 2m\lambda$  before it vanishes asymptotically. The position of the peak is almost independent of  $\delta$  but does depend on  $\lambda$ . Since  $q/(2m\lambda) = L_{so}/\Lambda$ , the spin Hall current is maximum when  $\Lambda \approx L_{so}$ . In the limit of small disorder broadening, i.e., for large  $\delta$ ,  $\sigma_{yx}^s(q)$  resonates exactly at  $q = 2m\lambda$  as in the case of  $\delta = 1.2$ . Choosing different values of  $\delta$  for a fixed value of  $\Delta$  implies different values of  $L_{so}$ . The larger value of  $\delta$  means smaller  $L_{so}$ . As the value of  $L_{so}$  becomes smaller, the decrease in SHC will be faster from its peak value for both increase and decrease of  $\Lambda$ . This is the reason for narrower width of the SHC peak for larger values of  $\delta$  as we see in Fig. 2. The peak value of  $\sigma_{yx}^s(q)$  is larger for larger  $\delta$ , i.e., for larger  $\lambda$  as well as  $\tau$ .

To demonstrate the resonance in  $\sigma_{yx}^s(0, q)$  analytically, we calculate  $\hat{j}_x^s$  in Eq. (3) for  $q_x = 0$  and  $q_y = q$ . We sum over infinite series of ladder diagrams starting with the contribution from ladder with just one bar

$$\frac{1}{m\tau} \int \frac{d\mathbf{k}'}{(2\pi)^2} \hat{G}_{\mathbf{k}'}^A(0) \hat{j}_x^s \left( \mathbf{k}' + \frac{\mathbf{q}}{2} \right) \hat{G}_{\mathbf{k}'+\mathbf{q}}^R(0) \equiv \sum_{\alpha=0}^3 J_{\alpha}^0 \sigma_{\alpha}. \quad (5)$$

Expressing  $\hat{j}_x^s = \sum_{\alpha=0}^3 \mathcal{J}_{\alpha} \sigma_{\alpha}$  and summing over geometrical series obtained from ladder diagrams we find that only  $\mathcal{J}_2$  and  $\mathcal{J}_3$  survive at  $q_x = 0$  and they are

$$\mathcal{J}_2 \approx J_2^0 \left[ \frac{1}{1 - I_{22}} + \frac{I_{23} I_{32}}{(1 - I_{22})^2 (1 - I_{33})} \right] + J_3^0 \frac{I_{23}}{(1 - I_{22})(1 - I_{33})}, \quad (6)$$

$$\mathcal{J}_3 \approx J_3^0 \frac{1}{1 - I_{33}} + J_2^0 \frac{I_{32}}{(1 - I_{22})(1 - I_{33})}, \quad (7)$$

where

$$I_{\alpha\beta} = \frac{1}{2m\tau} \text{Tr} \left[ \int \frac{d\mathbf{k}'}{(2\pi)^2} \sigma_{\alpha} \hat{G}_{\mathbf{k}'}^A(0) \sigma_{\beta} \hat{G}_{\mathbf{k}'+\mathbf{q}}^R(0) \right]. \quad (8)$$

Since the resonance occurs in  $\sigma_{yx}^s(0, q)$  at  $q \sim 2m\lambda \ll k_F$ , we may wish to evaluate  $I_{\alpha\beta}$  up to quadratic in  $q$  and hence the relevant components are  $I_{22} = 1 - (\Delta/m)\tau q^2 - \delta^2/2$ ,  $I_{33} = 1 - (\Delta/m)\tau q^2 - \delta^2$ , and  $I_{32} = -I_{23} = 2i\delta\Delta q/k_F$  for  $\delta \ll 1$ . In this approximation,  $J_2^0 = e\lambda\delta^2/2$  and  $J_3^0 = -iqe\delta^2/4m$ . Using Eq. (2), we thus find

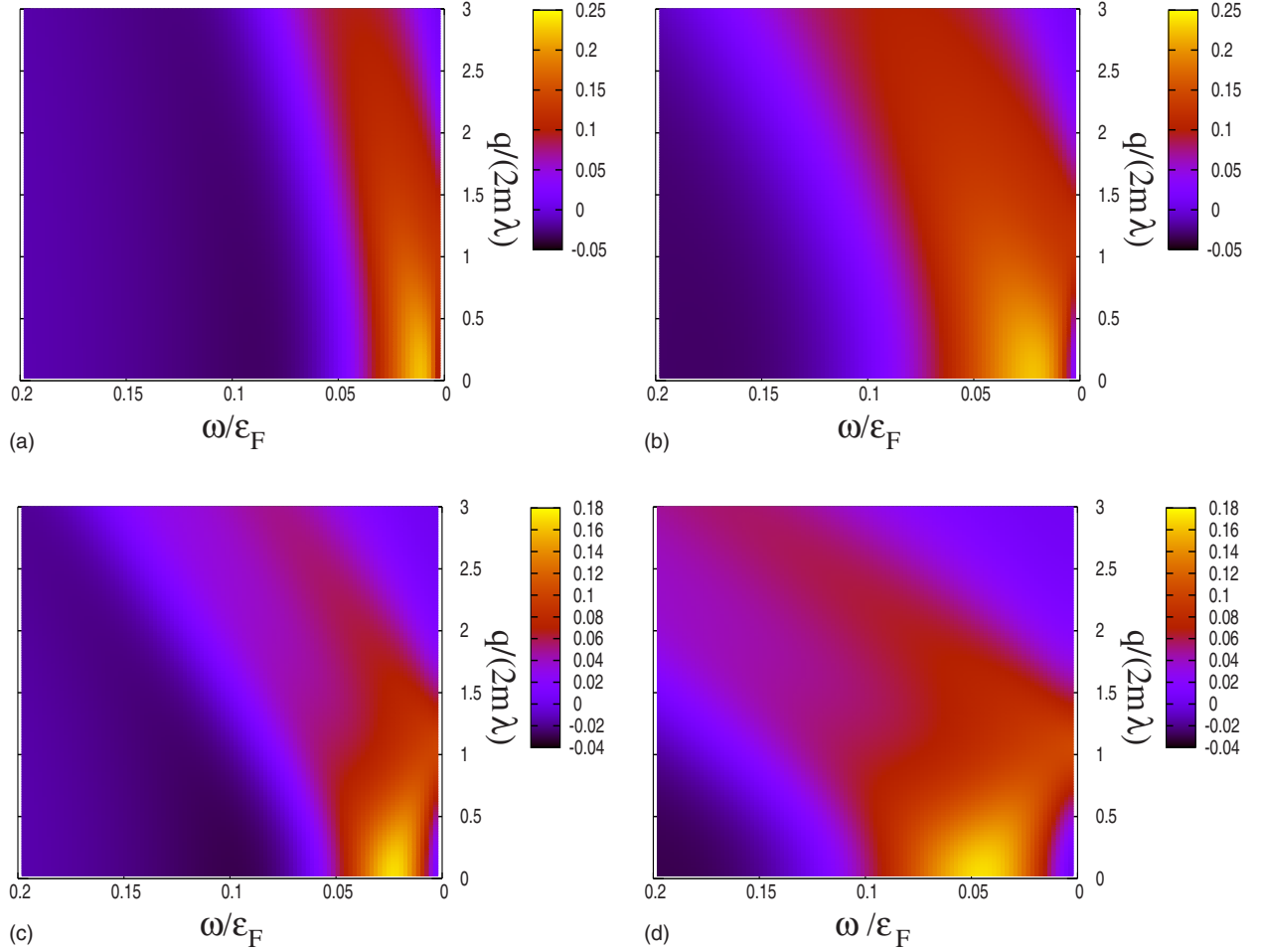


FIG. 1. (Color online) Transverse spin Hall conductivity  $\sigma_{yx}^s$  in the unit of  $\delta^2(e/2\pi)$  as a function of  $q/(2m\lambda)$  (vertical axis) and  $\omega/\epsilon_F$  (horizontal axis). The parameters  $\Delta=10$ ,  $\delta=0.4$  (b),  $\Delta=20$ ,  $\delta=0.4$  (d),  $\Delta=10$ ,  $\delta=0.8$  (a), and  $\Delta=20$ ,  $\delta=0.8$  (c) are considered.

$$\sigma_{yx}^s(0, q) \approx \frac{(e/2\pi)\delta^2\tilde{q}^2}{(\tilde{q}^2+1)(\tilde{q}^2+2)} \left[ \frac{5}{4} - \frac{1}{(\tilde{q}^2+1)} \right], \quad (9)$$

where  $\tilde{q}=q/(2m\lambda)$ . As we have seen in our numerical evaluation,  $\sigma_{yx}^s$  is independent of  $\Delta$  and proportional to  $\delta^2$  for small  $\delta$ . The expression of  $\sigma_{yx}^s$  [Eq. (9)] is graphically shown in Fig. 2. It agrees well with the numerical evaluation at low  $\tilde{q}$ . The discrepancy at higher  $\tilde{q}$  is expected as we have evaluated  $I_{\alpha\beta}$ ,  $J_2^0$ , and  $J_3^0$  analytically up to quadratic in  $q$  only. Nevertheless analytical expression (9) explicitly shows the resonance in  $\sigma_{yx}^s(0, q)$ .

### III. SPIN HALL CURRENT FOR SPIN TORQUE

The above calculation of  $\sigma_{yx}^s$  has been performed using the conventional definition of the spin current. We now consider the new definition of the spin current  $\mathcal{J}^3$  which is defined to satisfy continuity equation  $\partial_t S_3 + \nabla \cdot \mathcal{J}^3 = 0$  and can be expressed as the sum of  $\mathbf{J}^3$  and spin torque dipole density  $\mathbf{P}^\tau$ ; i.e.,  $\mathcal{J}^3 = \mathbf{J}^3 + \mathbf{P}^\tau$  as proposed by Shi *et al.*<sup>29</sup> Here  $S_3$  and  $\mathbf{J}^3$  are the spin density and conventional spin current-density operators, respectively. Further the spin torque density operator is expressed as  $\tau_3(\mathbf{r}) = -\nabla \cdot \mathbf{P}^\tau(\mathbf{r})$  since the average torque den-

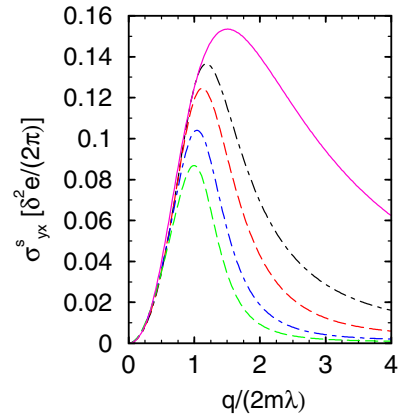


FIG. 2. (Color online) Spin Hall conductivity  $\sigma_{yx}^s$  (dot-dashed and dashed lines) for the conventional definition of the spin current vs  $q/(2m\lambda)$  for  $\delta=0.1, 0.4, 0.8$ , and  $1.2$  from top to bottom and for a fixed value of  $\Delta=10$ .  $\sigma_{yx}^s$  is in the unit of  $e\delta^2/2\pi$ .  $\sigma_{yx}^s$  is indeed independent of  $\Delta$  as we have checked for a wide range of  $\Delta > 1$ . The maximum value of  $\sigma_{yx}^s$  occurs at  $q/(2m\lambda) \approx 1$ . The width of the peak in  $\sigma_{yx}^s$  is larger for smaller values of  $\delta$  and the value of the peak is larger for larger values of  $\delta$ . Solid line represents the analytical expression (9) of  $\sigma_{yx}^s$  for small  $\delta$ .

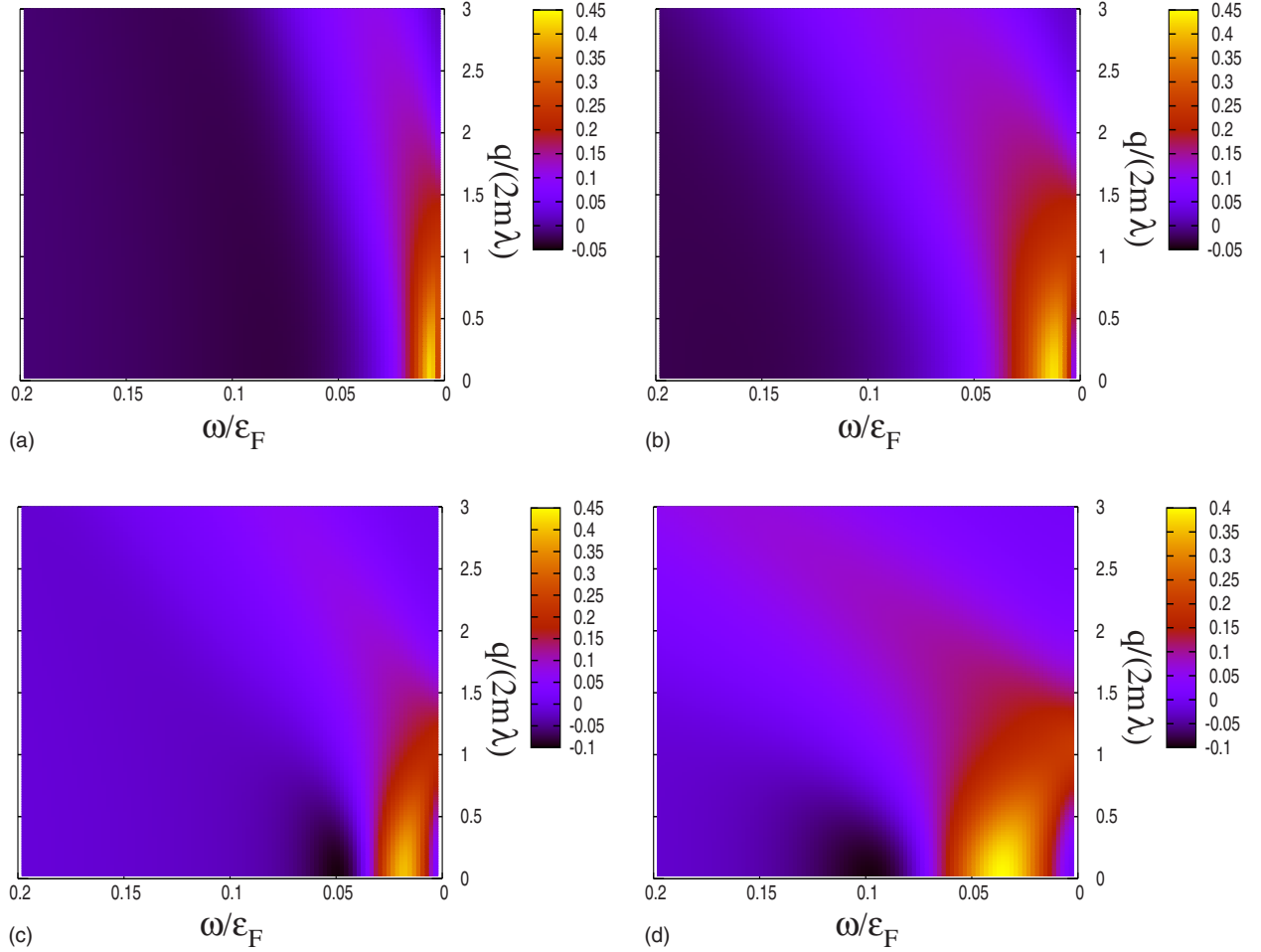


FIG. 3. (Color online) Total transverse spin Hall conductivity  $\sigma_{yx}^{\text{cs}}$  in the unit of  $\delta^2(e/2\pi)$  as a function of  $q/(2m\lambda)$  (vertical axis) and  $\omega/\epsilon_F$  (horizontal axis). The parameters  $\Delta=10$ ,  $\delta=0.4$  (b),  $\Delta=20$ ,  $\delta=0.4$  (d),  $\Delta=10$ ,  $\delta=0.8$  (a), and  $\Delta=20$ ,  $\delta=0.8$  (c) are considered.

sity vanishes in the bulk of the system. The second-quantized form of the spin torque is

$$\tau_3(\mathbf{q}, t) = \sum_{\mathbf{k}, \alpha, \beta} C_{\mathbf{k}, \alpha}^\dagger(t) \hat{\tau}_{\alpha\beta} \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) C_{\mathbf{k}+\mathbf{q}, \beta}(t), \quad (10)$$

where  $C_{\mathbf{k}, \alpha}^\dagger(t)$  is the electronic creation operator of momentum  $\mathbf{k}$  and spin  $\alpha$  (up or down) at time  $t$  and the spin torque operator  $\hat{\tau}(\mathbf{k} + \frac{\mathbf{q}}{2}) = \lambda(\mathbf{k} + \frac{\mathbf{q}}{2}) \cdot \boldsymbol{\sigma}$ . We define spin torque-charge current correlation function

$$Q_x^{\tau_0}(q_x, q_y, \omega) = \frac{1}{2\pi} \text{Tr} \left[ \int \frac{d\mathbf{k}}{(2\pi)^2} \hat{\tau} \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) \hat{G}_{\mathbf{k}}^A(0) \times \left\{ \hat{j}_x^0 \left( \mathbf{k} + \frac{\mathbf{q}}{2} \right) + \hat{j}_x^0(\mathbf{q}) \right\} \hat{G}_{\mathbf{k}+\mathbf{q}}^R(\omega) \right], \quad (11)$$

such that  $\tau_3(0, q, \omega) = Q_x^{\tau_0}(0, q, \omega) E_x$ . Therefore the extra part of transverse spin Hall conductivity for an application of electric field along  $x$  direction is

$$\sigma_{yx}^{(2)}(q, \omega) = - \frac{\Im Q_x^{\tau_0}(q, \omega)}{q}. \quad (12)$$

The total (conserved) transverse SHC is then  $\sigma_{yx}^{\text{cs}}(q, \omega) = \sigma_{yx}^s(q, \omega) + \sigma_{yx}^{(2)}(q, \omega)$ . The values of  $\sigma_{yx}^{\text{cs}}(q, \omega)$  for  $\Delta=10$  and  $20$  and  $\delta=0.4$  and  $0.8$  are shown in Fig. 3. The total transverse SHC is qualitatively similar to the same calculated for conventional spin current shown in Fig. 1. Figure 4 shows  $\sigma_{yx}^{\text{cs}}(q)$  at zero frequency for  $\Delta=10$  and  $\delta=0.1, 0.4, 0.8$ , and  $1.2$ , although the value of  $\Delta$  is immaterial. The only relevant parameter is  $\delta$ . The peak in SHC occurs at  $q/(2m\lambda) \approx 1$  and the width of the peak is larger for smaller value of  $\delta$  as in the previous case shown in Fig. 2. Also the maximum value of  $\sigma_{yx}^{\text{cs}}(q)$  is larger for larger  $\delta$  and it is almost two times that of  $\sigma_{yx}^s(q)$ .

#### IV. MECHANISM FOR ANOMALOUS SPIN HALL CURRENT

The time derivative of the charge current operator is given by

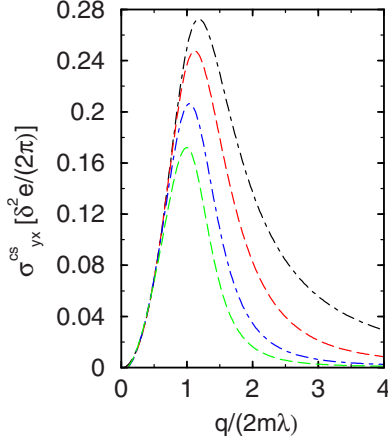


FIG. 4. (Color online) Total spin Hall conductivity  $\sigma_{sh}^{cs}(q)$  as a function of  $q/(2m\lambda)$ . Other quantities and descriptions are the same as in Fig. 2.

$$\partial_{\mathbf{j}\mathbf{k}} \hat{\mathbf{j}}_{\mathbf{k}}^0 = \frac{e}{m} \partial_{\mathbf{k}} \mathbf{k} - 4em\lambda^2 (\mathbf{e}_z \times \hat{\mathbf{j}}_{\mathbf{k}}^3). \quad (13)$$

Therefore in the presence of external electric field  $\mathbf{E}$ , the steady-state equation becomes

$$\frac{e^2}{m} \mathbf{E} = \frac{\langle \hat{\mathbf{j}}_{\mathbf{k}}^0 \rangle}{\tau} + 4em\lambda^2 (\mathbf{e}_z \times \langle \hat{\mathbf{j}}_{\mathbf{k}}^3 \rangle), \quad (14)$$

where  $\mathbf{e}_z$  is the unit vector along  $z$  direction and angular brackets represent average value of the quantity inside angular brackets. Therefore the electric field creates not only the charge current but also the spin current for an electron with momentum  $\mathbf{k}$ . Although some electrons move perpendicular to the electric field, their spin precesses about  $\Omega_{\mathbf{k}} = (k_y \mathbf{e}_x - k_x \mathbf{e}_y) / |\mathbf{k}|$  while moving and hence no net spin current flows in the bulk of the system. This is an alternative description for vanishing  $\sigma_{yx}^s$  in the system for static and uniform electric fields. However the situation alters when the electric field is static but nonuniform along its transverse direction. The basic physics behind the anomalous behavior for spatial variation of the electric field is described in Fig. 5. If the electric field is applied along  $x$  direction, due to the wave propagation along  $y$  direction, sign of the electric field changes alternately along  $y$  direction with the wavelength  $\Lambda = 2\pi/q$ . The spin of an electron moving along  $y$  direction will precess in the  $y-z$  plane with the precession length  $L_{so}$ . However at the position where the sign of the electric field changes, the clockwise (counterclockwise) precession will change into counterclockwise (clockwise) precession as in Fig. 5. This is because the sign of the  $z$  component of the spin changes as described by Eq. (14). The mode of spin precession for the electrons moving along negative  $y$  direction will be exactly opposite and therefore there will be a net  $z$ -polarized spin-current along  $y$  direction. The spin current will be maximum when  $L_{so} \approx \Lambda$  and it will sharply fall for the change in  $\Lambda$  either way. This argument for *anomalous* spin Hall current is somewhat similar to the ineffectiveness concept of Pippard<sup>33</sup> for anomalous skin effect in metals but the fundamental difference is that the former occurs for  $\Lambda \approx L_{so}$  while the latter

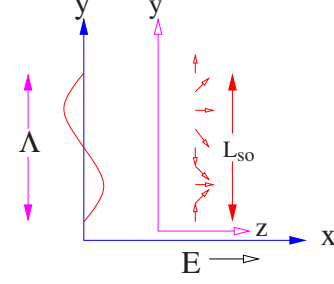


FIG. 5. (Color online) Electric field along  $x$  axis and its variation along  $y$  axis with wavelength  $\Lambda$ . Electrons moving along  $y$  axis will have spin precession in the  $y-z$  plane.  $L_{so}$  is the spin precession length which is the length traversed by an electron while its spin precesses by an angle  $2\pi$ . Lines with arrow indicate the direction of spin while it precesses. The mode of rotation of spin changes when the sign of the electric field changes. Therefore the spatial sign change of the electric field induces a net out-of-plane spin current in the transverse direction. The spin current will be at maximum when  $\Lambda \approx L_{so}$ .

is due to  $\Lambda \ll \ell$  ( $\ell$  is the mean-free path of an electron). The present picture is of course valid for  $\ell < L_{so}$ .

We have determined the transverse wave number  $q$  dependent spin Hall conductivity by using Kubo formula in Secs. II and III and show here how the basic physics presented in Fig. 5 describes the anomalous spin Hall current. On the other hand, there will be no longitudinal spin Hall current (when  $\mathbf{q} \parallel \mathbf{E}$ ) for nonuniform but static electric field because the electrons moving transverse to the electric field will not feel any change in sign of the electric field.

## V. DISCUSSION AND SUMMARY

Spin accumulation observed by Sih *et al.*<sup>4</sup> in two-dimensional electron gas corresponds to the value of  $\delta \sim 0.1$ . This is in the large disorder limit. Although the applied electric field is uniform, the electronic inhomogeneity in the system due to disorder may cause spatial variation of the electric field in the system. The variation of the electric field describes the presence of modes  $\mathbf{q}$ . If transverse  $q$  is closer to  $2m\lambda$ , the contribution of these modes to the spin Hall conductivity is not negligible. The presence of anomalous spin Hall current is then certain although the magnitude may be small since  $\delta$  is small in this experiment.<sup>4</sup> This contribution is intrinsic because the spin-orbit interaction is not disordered. Consideration of extrinsic mechanisms along with the intrinsic spin-orbit interaction provides finite<sup>34,35</sup> spin Hall conductivity in presence of uniform electric field as well. However, any quantitative comparison of the anomalous spin Hall conductivity presented in this paper to the experiment,<sup>4</sup> with the contribution arising from extrinsic effect,<sup>34,35</sup> or to the spin accumulation across the edges in a ballistic system<sup>36</sup> is beyond the scope of the present study. Nevertheless our theory may be tested by applying a spatially varying electric field with the variation along its transverse direction. The geometry for an experimental proposal to test the mechanism for “anomalous” spin Hall current is depicted in Fig. 6.

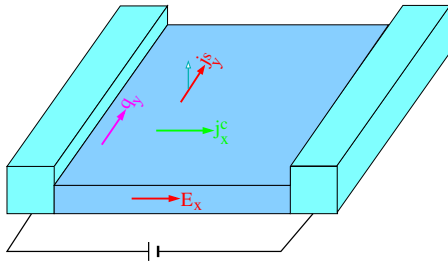


FIG. 6. (Color online) In a spin Hall bar geometry, an electric current  $j_x^c$  is driven through a 2DEG contacted with the metallic leads connected to a voltage source. The electric field  $E_x$  with transverse momentum  $q_y$  induces a nonvanishing bulk spin Hall current  $j_y^s$  with spin pointing perpendicular to the plane of 2DEG.

The other studies at finite  $q$  in this Rashba 2DEG are the induction of spin-density by electromagnetic wave,<sup>37</sup> the response of the in-plane polarization<sup>38</sup> to the transverse electric field in a pure system, and determination of density-density correlation function<sup>39</sup> at all  $q$ . In a cubic Rashba model which is relevant for two-dimensional hole gas, the intrinsic SHC is nonzero,<sup>16</sup> but the conserved spin Hall conductivity

vanishes<sup>27</sup> for short-ranged impurity potential. Therefore it is indeed interesting to look into if these systems also have anomalous spin Hall current<sup>40</sup> like what we have described here.

In summary, we have determined spin Hall conductivity at finite frequency and finite transverse wave vector in a disordered two-dimensional electron gas with Rashba spin-orbit interaction. Interestingly at zero or small frequencies, we have found an anomalous spin Hall conductivity which resonates when the wavelength of the spatial variation of the electric field matches with the length of spin precession. The mechanism responsible for this is the change in the direction of spin precession for electrons moving perpendicular to the electric field when the sign of the electric field changes due to spatial variation. This is primarily due to the change in sign of the out-of-plane component of spin.

#### ACKNOWLEDGMENTS

The authors thank J. K. Bhattacharjee, T. V. Ramakrishnan, and K. Ray for stimulating discussions and M. Pletyukhov for sharing his calculation before publication.

\*Present address: Department of Physics, Sukanta Mahavidyalaya, Jalpaiguri, 735 210 West Bengal, India.

<sup>1</sup>*Semiconductor Spintronics and Quantum Computation*, edited by D. D. Awschalom, D. Loss, and N. Samarth (Springer, Hiedelberg, 2002).

<sup>2</sup>Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, *Science* **306**, 1910 (2004).

<sup>3</sup>J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, *Phys. Rev. Lett.* **94**, 047204 (2005).

<sup>4</sup>V. Sih, R. C. Myers, Y. K. Katao, W. H. Lau, A. C. Gossard, and D. D. Awschalom, *Nat. Phys.* **1**, 31 (2005).

<sup>5</sup>M. I. Dyakonov and V. I. Perrel, *Phys. Lett.* **35A**, 459 (1971).

<sup>6</sup>J. E. Hirsch, *Phys. Rev. Lett.* **83**, 1834 (1999).

<sup>7</sup>S. Zhang, *Phys. Rev. Lett.* **85**, 393 (2000).

<sup>8</sup>J. Smit, *Physica (Amsterdam)* **21**, 877 (1955).

<sup>9</sup>L. Berger, *Phys. Rev. B* **2**, 4559 (1970).

<sup>10</sup>R. Karplus and J. M. Luttinger, *Phys. Rev.* **95**, 1154 (1954).

<sup>11</sup>S. Murakami, N. Nagaosa, and S. C. Zhang, *Science* **301**, 1348 (2003).

<sup>12</sup>J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, *Phys. Rev. Lett.* **92**, 126603 (2004).

<sup>13</sup>S. O. Valenzuela and M. Tinkham, *Nature (London)* **442**, 176 (2006).

<sup>14</sup>H. A. Engel, B. I. Halperin, and E. I. Rashba, *Phys. Rev. Lett.* **95**, 166605 (2005).

<sup>15</sup>W.-K. Tse and S. Das Sarma, *Phys. Rev. Lett.* **96**, 056601 (2006).

<sup>16</sup>B. A. Bernevig and S. C. Zhang, *Phys. Rev. Lett.* **95**, 016801 (2005); S. Y. Liu and X. L. Lei, *Phys. Rev. B* **72**, 155314 (2005).

<sup>17</sup>E. I. Rashba, *Sov. Phys. Solid State* **2**, 1109 (1968).

<sup>18</sup>E. I. Rashba, *Phys. Rev. B* **68**, 241315(R) (2003); **70**,

201309(R) (2004).

<sup>19</sup>J. I. Inoue, G. E. W. Bauer, and L. W. Molenkamp, *Phys. Rev. B* **70**, 041303(R) (2004).

<sup>20</sup>E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, *Phys. Rev. Lett.* **93**, 226602 (2004).

<sup>21</sup>S. Y. Liu, X. L. Lei, and N. J. M. Horing, *Phys. Rev. B* **73**, 035323 (2006).

<sup>22</sup>O. V. Dimitrova, *Phys. Rev. B* **71**, 245327 (2005); O. Chalaev and D. Loss, *ibid.* **71**, 245318 (2005).

<sup>23</sup>R. Raimondi and P. Schwab, *Phys. Rev. B* **71**, 033311 (2005).

<sup>24</sup>A. Khaetskii, *Phys. Rev. Lett.* **96**, 056602 (2006).

<sup>25</sup>D. N. Sheng, L. Sheng, Z. Y. Weng, and F. D. M. Haldane, *Phys. Rev. B* **72**, 153307 (2005).

<sup>26</sup>K. Nomura, J. Sinova, N. A. Sinitsyn, and A. H. MacDonald, *Phys. Rev. B* **72**, 165316 (2005).

<sup>27</sup>N. Sugimoto, S. Onoda, S. Murakami, and N. Nagaosa, *Phys. Rev. B* **73**, 113305 (2006).

<sup>28</sup>A. Sensharma and S. S. Mandal, *J. Phys.: Condens. Matter* **18**, 7349 (2006).

<sup>29</sup>J. Shi, P. Zhang, D. Xiao, and Q. Niu, *Phys. Rev. Lett.* **96**, 076604 (2006).

<sup>30</sup>Q. Lin, S. Y. Liu, and X. L. Lei, *Appl. Phys. Lett.* **88**, 122105 (2006); H. A. Engel, E. I. Rashba, and B. I. Halperin, *Phys. Rev. Lett.* **98**, 036602 (2007); M. Milletari, R. Raimondi, and P. Schwab, *EPL* **82**, 67005 (2008).

<sup>31</sup>P. Lucignano, R. Raimondi, and A. Tagliacozzo, *Phys. Rev. B* **78**, 035336 (2008).

<sup>32</sup>See, for example, G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1990).

<sup>33</sup>A. B. Pippard, *Philos. Trans. R. Soc. London, Ser. A* **250**, 325 (1957).

<sup>34</sup>W.-K. Tse and S. Das Sarma, *Phys. Rev. B* **74**, 245309 (2006).

<sup>35</sup>E. M. Hankiewicz, G. Vignale, and M. E. Flatte, Phys. Rev. Lett. **97**, 266601 (2006).

<sup>36</sup>V. A. Zyuzin, P. G. Silvestrov, and E. G. Mishchenko, Phys. Rev. Lett. **99**, 106601 (2007).

<sup>37</sup>A. Shnirman and I. Martin, Europhys. Lett. **78**, 27001 (2007).

<sup>38</sup>E. Rashba, J. Supercond. **18**, 137 (2005).

<sup>39</sup>M. Pletyukhov and S. Konschuh, arXiv:0705.2419 (unpublished).

<sup>40</sup>A. Sensharma and S. S. Mandal (unpublished).